Dynamics of Degenerate Electron Gas in a Magnetic Field*

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Some aspects of the correlations between interacting electrons in a magnetic field are studied by a quantum kinetic approach, taking into account the quantization of the orbital motion of the electrons. The treatment is based on the introduction of an hierarchy of equations of motion for the s-body density matrices and employing a decoupling scheme, valid in the self-consistent field approximation. In this way the dielectric function is introduced "naturally," and the response of the system to an applied field is calculated. The dielectric function reflects the two main effects of the correlations, i.e., the collective excitations of the system and the dynamical shielding. The dielectric function plays a role also in the "dressing" of the interacting electrons. This is shown by calculating the spectrum of the density fluctuations and the equation which governs the approach to equilibrium of the system. The method is not limited to systems in thermal equilibrium, but also applies to systems in other stationary equilibrium states.

I. INTRODUCTION

THE properties of a system of noninteracting degenerate electrons in strong uniform magnetic fields have been studied by many authors.[1,2] The main features of this system are the oscillations of the thermodynamic quantities in the magnetic field. The novel effect of the (strong) magnetic field on the electrons is the quantization of the orbital motion. The effect of the orbital quantization is important, when the energy associated with this quantization is of the same order or larger than the average kinetic energy per particle. That is, when $\hbar\omega_c \gtrsim E_F$, where ω_c is the cyclotron frequency of the particle and E_F is the Fermi energy. The question arises as to how the presence of the interaction between the electrons would affect the various properties of the system in magnetic fields.

Recently, attempts were made to include the Coulomb interaction and to study its influence. The case of the Fermi liquid model in thermal equilibrium was studied by Bychkov and Gorkov[3] and others[4] neglecting the long-range part of the Coulomb interaction. Some of the properties of the ground state and low-lying excitations of an interaction electron gas were studied by Kohn.[5] Some properties of the interacting electron gas in thermal equilibrium, especially the plasma oscillations in magnetic field and their effects, were investigated by Stephen,[6] Akhiezer[7] and Zyryanov.[8] Stephen[6] approached the problem of plasma oscillations by employing a technique, which is originally due to Montroll and Ward,[9] and made connections with the classical treatment of plasma oscillations. Akhiezer[7] generalized former work on the energy loss of a test particle to the case of strong magnetic fields by the method of temperature dependent Green's function technique. Zyryanov[8] adopted the self-consistent field approach of Ehrenreich and Cohen[10] to derive the dispersion relations for plasma oscillations in high magnetic fields, but made some unnecessary approximations in his calculation.[10a]

The present paper is concerned with some aspects of the Coulomb interaction between electrons in a strong uniform magnetic field. The treatment of the interaction is restricted to the domain of validity of the random phase approximation[11] or the self-consistent field approach (SCF).[10] The magnetic field is assumed to be strong enough, so that the quantization of the orbital motion of the electrons takes place. The magnetic interactions and retardation of the Coulomb interactions between the electrons are neglected, and a homogeneous positive background is assumed for average neutrality. The method of approach is a generalization of the well known one, employing in the investigations of classical systems of interacting electrons, namely. the kinetic approach. This quantum kinetic method, which was developed in a previous paper[12] (hereafter referred to as

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[9] E. W. Montroll and J. Ward, Phys. Fluids **1**, 51 (1958).
[10] H. Ehrenrich and M. H. Cohen, Phys. Rev. **115**, 786 (1959).
[10a] It has been brought to our attention that John J. Quinn and Sergio Rodriguez [Phys. Rev. **128**, 2487 (1962)] have been investigating the same problem.
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I), is a suitable one for the study of both thermal equilibrium and quasistationary equilibrium.

In Sec. II we recall the formal development of the hierarchy of equations for the density matrix, as was derived in I. The truncated set of equations, for the one-electron density matrix and the two-electron correlation function, is given in the SCF approximation. A proper representation in terms of the "Landau states" is introduced to take account of the quantized orbital motion of the electrons in the magnetic field. In Sec. III we present calculations of the dielectric function, dielectric tensor, and the density-fluctuation spectrum of the system. These functions are calculated by the kinetic method for the case, where the electrons are known to be distributed among the "Landau states" in a given "diagonal" distribution function. (Fermi distribution is the one which corresponds to the special case of thermal equilibrium.) A concept of a "dressed" electron is introduced in a natural manner, to account the the cloud of the Landau electrons, associated with a given "test electron" due to the Coulomb interaction in the SCF approximation (compare with Rostoker[13] and Ron[14]). The spectrum of density fluctuation is obtained by virtue of an ensemble average of free dressed electrons. Section IV deals with the derivation of an irreversible kinetic equation for the Landau electrons. A method developed in I, which is similar to the one originally used by Dupree,[15] is adopted here, with slight modification, to solve the integral equation for the correlation function. The kinetic equation governs the approach to thermal equilibrium of the Landau electrons by virtue of both the individual and collective aspects of their interaction. We conclude our paper with a brief discussion of the results (Sec. V).

II. HAMILTONIAN FORMULATION

We consider a system of N electrons in a unit volume with a mass m and a charge $+e$. The electrons interact with each other via Coulomb potential $v(\mathbf{r})=e^2/r$ and move in a constant magnetic field $\mathbf{B}$. A neutralizing positive background is assumed. In the second quantization representation the Hamiltonian of the system reads (with $\hbar=1$)

$$H=\frac{1}{2m}\sum_{\sigma}\int d\mathbf{r}\psi_{\sigma}^{\dagger}(\mathbf{r})\left[-i\frac{\partial}{\partial\mathbf{r}}-\frac{e}{c}\mathbf{A}(\mathbf{r})\right]\psi_{\sigma}(\mathbf{r})$$
$$+\tfrac{1}{2}\sum_{\sigma\sigma'}\int d\mathbf{r}d\mathbf{r}'\psi_{\sigma}^{\dagger}(\mathbf{r})\psi_{\sigma'}^{\dagger}(\mathbf{r}')v(\mathbf{r}-\mathbf{r}')\psi_{\sigma'}(\mathbf{r}')\psi_{\sigma}(\mathbf{r}), \quad (1)$$

where $\psi_{\sigma}^{\dagger}(\mathbf{r})$ and $\psi_{\sigma}(\mathbf{r})$ are, respectively, the creation and annihilation operators of the electrons with spin σ and position $\mathbf{r}$, satisfying the usual anticommutation relation, and $\mathbf{A}(\mathbf{r})$ is the vector potential corresponding to the uniform $\mathbf{B}$ field. From here on, our formal development proceeds in close analogy with that of I, and for completeness we shall give brief review of it.

The one-electron density matrix is denoted by

$$F_{\sigma_1}(1,1')=F_{\sigma_1}(\mathbf{r}_1,\mathbf{r}_1',t)$$
$$=\mathrm{Tr}\{D\psi_{\sigma_1}^{\dagger}(\mathbf{r}_1',t)\psi_{\sigma_1}(\mathbf{r}_1,t)\}$$
$$=\langle\psi_{\sigma_1}^{\dagger}(1')\psi_{\sigma_1}(1)\rangle, \quad (2)$$

where the Heisenberg representation for the operators is used, and D is the time-independent density matrix of the whole system. The two-electron density matrix is given by

$$F_{\sigma_1\sigma_2}(1,2;1',2')=\langle\psi_{\sigma_1}^{\dagger}(1')\psi_{\sigma_2}^{\dagger}(2')\psi_{\sigma_2}(2)\psi_{\sigma_1}(1)\rangle, \quad (3)$$

and higher order functions may be defined in the same manner. The equations of motion of the F's are

$$\left(i\frac{\partial}{\partial t}+T_1\right)F_{\sigma_1}(1,1')=\sum_{\sigma_3}\int d\mathbf{r}_2\bar{W}_{12}F_{\sigma_1\sigma_2}(1.2;1',2), \quad (4)$$

and

$$\left(i\frac{\partial}{\partial t}+T_1+T_2-W_{12}\right)F_{\sigma_1\sigma_2}(1,2;1',2')$$
$$=\sum_{\sigma_3}\int d\mathbf{r}_3(\bar{W}_{13}+\bar{W}_{23})F_{\sigma_1\sigma_2\sigma_3}(1,2,3;1',2',3)\ \text{etc.}, \quad (5)$$

where T_i, W_{ij}, and $\bar{W}_{ij}$ stand for

$$T_i=\frac{1}{2m}\left\{-\left[-i\frac{\partial}{\partial\mathbf{r}_i}-\frac{e}{c}\mathbf{A}(\mathbf{r}_i)\right]^2\right.$$
$$\left.+\left[+i\frac{\partial}{\partial\mathbf{r}_i'}-\frac{e}{c}\mathbf{A}(\mathbf{r}_i')\right]\right\}, \quad (6)$$

$$W_{ij}=v(\mathbf{r}_i-\mathbf{r}_j)-v(\mathbf{r}_i'-\mathbf{r}_j'), \quad (7a)$$

and

$$\bar{W}_{ij}=v(\mathbf{r}_i-\mathbf{r}_j)-v(\mathbf{r}_i'-\mathbf{r}_j). \quad (7b)$$

If Eqs. (4) and (5) use has been made of the equations on motion for the ψ operators

$$i\frac{\partial}{\partial t}\psi_{\sigma}(\mathbf{r})=\left[-i\frac{\partial}{\partial\mathbf{r}}-\frac{e}{c}\mathbf{A}(\mathbf{r})\right]^2\psi_{\sigma}(\mathbf{r})$$
$$+\sum_{\sigma'}\int d\mathbf{r}'v(\mathbf{r}-\mathbf{r}')\psi_{\sigma'}^{\dagger}(\mathbf{r}')\psi_{\sigma'}(\mathbf{r}')\psi_{\sigma}(\mathbf{r}) \quad (8)$$

and a similar one for $\psi^{\dagger}$.

We now introduce a proper antisymmetrization operator

$$\gamma_n=\prod_{j=2}^{n}[1-\sum_{k=1}^{j=1}\delta_{\sigma_j\sigma_k}P_{jk}], \quad (9)$$

[13] N. Rostoker, Phys. Fluids **3**, 922 (1960), and Nucl. Fusion **1**, 101 (1961).

[14] A. Ron, Phys. Rev. **132**, 978 (1963).

[15] T. Dupree, Phys. Fluids **4**, 696 (1961). See also P. A. Wolff, Phys. Fluids **5**, 316 (1962).

where P_{jk} permutes the $\mathbf{r}_j$ and r_k, and define new functions by means of

$$F_{\sigma_1\cdots\sigma_n}(1,\cdots n;1'\cdots n') = \gamma_n f_{\sigma_1\cdots\sigma_n}(1\cdots n;1'\cdots n'). \quad (10)$$

We also use the truncation scheme of I, which is originally due to Bogoliubov, by introducing a genuine two-electron correlation function

$$g_{\sigma_1\sigma_2}(1,2;1',2') = f_{\sigma_1\sigma_2}(1,2;1',2') - f_{\sigma_1}(1,1')f_{\sigma_2}(2,2'), \quad (11)$$

and approximating the three-electron function by

$$f_{\sigma_1\sigma_2\sigma_3}(1,2,3;1',2',3') = \prod_{i=1}^{3} f_{\sigma_i}(i,i') + \sum_{\text{pairs}} f_{\sigma_i}(i,i')g_{\sigma_j\sigma_k}(j,k;j',k'). \quad (12)$$

In the Coulomb case, under the self-consistent field approximation, we obtain the coupled set of equations

$$\left[i\frac{\partial}{\partial t} + T_1 - \sum_{\sigma_2}\int d\mathbf{r}_2 \bar{W}_{12} f_{\sigma_2}(2,2)\right] f_{\sigma_1} = -\int d\mathbf{r}_2 \bar{W}_{12} f_{\sigma_1}(2,1')f_{\sigma_2}(1,2) + \sum_{\sigma_2}\int d\mathbf{r}_2 \bar{W}_{12} g_{\sigma_1\sigma_2}(1,2;1',2), \quad (13)$$

and

$$\left[i\frac{\partial}{\partial t} + T_1 + T_2 - \sum_{\sigma_3}\int d\mathbf{r}_3(\bar{W}_{13}+\bar{W}_{23}) f_{\sigma_3}(3,3)\right] g_{\sigma_1\sigma_2}(1,2;1',2') - \sum_{\sigma_3}\int d\mathbf{r}_3[\bar{W}_{13} f_{\sigma_1}(1,1')g_{\sigma_2\sigma_3}(2,3;2,3) + \bar{W}_{23} f_{\sigma_2}(2,2')g_{\sigma_1\sigma_3}(1,3;1',3)]$$

$$= W_{12} f_{\sigma_1}(1,1')f_{\sigma_2}(2,2') - \int d\mathbf{r}_3[\bar{W}_{13} f_{\sigma_1}(1,1')f_{\sigma_2}(2,3)f_{\sigma_2}(3,2') + \bar{W}_{23} f_{\sigma_1}(1,3')f_{\sigma_2}(2,2')f_{\sigma_1}(3,1')]. \quad (14)$$

We wish to point out that the terms on the right-hand side of Eq. (13) are, in the self-consistent field approximation, small compared to those on the left-hand side. A detailed discussion of this subject was given by Guernsey (see also I).

To make further progress, we notice that our set of equations, Eqs. (13) and (14), is a complicated differential-integral equation in the configuration space, and a change of representation is called for. A proper representation is, undoubtedly, the so-called "Landau representation,[1,2]" where the electrons are first subjected to the magnetic field and then interact with each other. Introducing the Landau gauge, with $\mathbf{B}$ in the z direction,

$$\mathbf{A}(\mathbf{r}) \equiv \{0, Bx, 0\}, \quad (15)$$

the Schrödinger equation for an electron reads

$$-\frac{1}{2m}\left[\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} - i\frac{eB}{c}x\right)^2 + \frac{\partial^2}{\partial z^2}\right]\phi_\alpha(\mathbf{r}) = E_\alpha \phi_\alpha(\mathbf{r}). \quad (16)$$

Noticing that z and y are cyclic, we write

$$\phi_\alpha(\mathbf{r}) = \frac{1}{2\pi}\exp(ipz+iqy)u_\alpha(x), \quad (17)$$

and the equation for $u_\alpha(x)$ is

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + \frac{m}{2}\omega_c^2\left(x - \frac{q}{m\omega_c}\right)^2\right]u_\alpha(x) = \left(E_\alpha - \frac{p^2}{2m}\right)u_\alpha(x), \quad (18)$$

where the cyclotron frequency ω_c is

$$\omega_c = eB/mc. \quad (19)$$

Equation (18) is the wave equation for an harmonic oscillator with the energy eigenvalues

$$E_\alpha = \omega_c(n+\tfrac{1}{2}) + p^2/2m. \quad (20)$$

The eigenstates of Eq. (16) are

$$\phi_\alpha(\mathbf{r}) \equiv |\alpha\rangle = |n,p,q\rangle = (2\pi)^{-1}e^{ipz+iqy}\Phi_n(x - q/m\omega_c), \quad (21)$$

where Φ_n are the orthonormal set of eigenfunctions of the harmonic oscillator. The boundary conditions we impose on the wave function are the usual ones, namely, periodicity in the y and z directions and reflecting boundaries in the x direction (the border region may be neglected). The number of states belonging to given n and p (per unit volume, and regarding p as continuous) is

$$n_\alpha = m\omega_c/(2\pi)^2, \quad (22)$$

apart from spin. Finally, the proper representations for $\psi_\sigma(\mathbf{r})$ and $\psi_\sigma^\dagger(\mathbf{r})$ read

$$\psi_\sigma(\mathbf{r}) = \sum_\alpha \phi_\alpha(\mathbf{r})a_{\alpha,\sigma}, \quad (23a)$$

and

$$\psi_\sigma^\dagger(\mathbf{r}) = \sum_\alpha \phi_\alpha^*(\mathbf{r})a_{\alpha,\sigma}^\dagger, \quad (23b)$$

where the summation goes over all $\alpha \equiv n, p, q$, and $a_\alpha^\dagger$, a_α are, respectively, the creation and annihilation operators for the Landau states. In particular, we shall be interested in the distribution of the electrons among

the states α, that is, in

$$f_\sigma(\alpha)=\langle a_{\alpha,\sigma}^\dagger a_{\alpha,\sigma}\rangle=\int d\mathbf{r}_1 d\mathbf{r}_1' \phi_\alpha(\mathbf{r}_1')\phi_\alpha^*(\mathbf{r}_1) f_\sigma(1,1'), \quad (24)$$

with the normalization

$$N=\sum_{\sigma,\alpha} f_\sigma(\alpha). \quad (25)$$

III. RESPONSE FUNCTIONS

We consider now the system of electrons under the influence of a driving force. The assumption is made that the latter causes only small effects and that the system responds linearly.

A. The Dielectric Function

Before we turn to the calculation of the dielectric tensor in a magnetic field, it is convenient to derive, in a simpler way, the longitudinal component of this tensor, which is, sometimes denoted as the "dielectric function." We shall adopt this terminology in the present paper. The simple derivation of the dielectric function serves here just to illustrate the connection between the collective excitation of the system and the self-consistent field approach.

We consider only the left-hand side of Eq. (13) and write it as

$$\{i(\partial/\partial t)+T_1-[V(1)-V(1')]\}f_\sigma(1,1')=0, \quad (26)$$

where

$$V(1)=\sum_{\sigma_2}\int d\mathbf{r}_2 v(\mathbf{r}_1-\mathbf{r}_2) f_{\sigma_2}(2,2) \quad (27)$$

is the self-consistent field. If we now linearize Eqs. (26) and (27) around a "diagonal" distribution

$$f_\sigma^{(0)}(1,1')=\sum_\alpha \phi_\alpha^*(\mathbf{r}_1')\phi_\alpha(\mathbf{r}) f_\sigma^{(0)}(\alpha), \quad (28)$$

where $f_\sigma^{(0)}(\alpha)$ is assumed to depend only on n and p of α, we obtain

$$[i(\partial/\partial t)+E_{\alpha'}-E_\alpha]f_\sigma^{(1)}(\alpha,\alpha') = \sum_{\mathbf{k}} V_k\langle\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha'\rangle[f_\sigma^{(0)}(\alpha')-f_\sigma^{(0)}(\alpha)], \quad (29)$$

and

$$V_k=v_k\sum_{\alpha\alpha'\sigma}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle f_\sigma^{(1)}(\alpha,\alpha'). \quad (30)$$

In Eqs. (29) and (30) we have denoted by $f^{(1)}$ the perturbed density matrix (in the α representation), and by v_k the Fourier transform of $v(\mathbf{r})$,

$$v_k=\int d\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} v(\mathbf{r});$$

matrix elements of any function $F(\mathbf{r},\mathbf{p})$ of the particle operators we have abbreviated by

$$\langle\alpha'|F(\mathbf{r},\mathbf{p})|\alpha\rangle=\int d\mathbf{r}\phi_{\alpha'}^*(\mathbf{r})F\left(\mathbf{r},-i\frac{\partial}{\partial\mathbf{r}}\right)\phi_\alpha(\mathbf{r}). \quad (31)$$

We wish to point out that $f_\sigma^{(0)}(\alpha)$ is not the thermal equilibrium Fermi distribution, but any stationary distribution of the electrons in the n, p states. In particular, one can substitute for f^0 the Fermi distribution.

The solution of Eqs. (29) and (30) is straightforward. We perform Laplace transformations of both equations, substitute $f^{(1)}$ from Eq. (29) into Eq. (30), and discarding the initial $f^{(1)}$ we obtain

$$V_{\mathbf{k}}=v_k\sum_{\mathbf{k}'}V_{\mathbf{k}'}\sum_{\alpha\alpha'\sigma}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle \times\langle\alpha|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha'\rangle\frac{f_\sigma^{(0)}(\alpha')-f_\sigma^{(0)}(\alpha)}{E_{\alpha'}-E_\alpha-\omega-i\delta}, \quad (32)$$

where δ is a small positive convergence factor. Now, we notice that both $f_\sigma^0(\alpha)$ and E_α are independent of q, and therefore the summation

$$\sum_{qq'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle\langle\alpha|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha'\rangle$$

can be carried out first. We reserve the algebra for the Appendix and state only the result

$$\sum_{qq'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle\langle\alpha|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha\rangle = \frac{m\omega_c}{2\pi}\delta_{\mathbf{k}',\mathbf{k}}\delta_{p',p+k_\parallel}H_{nn'}\left[\left(\frac{2}{m\omega_c}\right)^{1/2}k_\perp\right], \quad (33)$$

where (compare to Akhiezer[7])

$$H_{nn'}(x)=\int_0^\infty ds I_0[x(s)^{1/2}]e^{-s}L_n(s)L_{n'}(s), \quad (34)$$

$$k_\parallel=\mathbf{k}\cdot\mathbf{B}/B, \quad k_\perp=\mathbf{k}\times\mathbf{B}/B; \quad (35)$$

$I_0(x)$ is the Bessel function of order zero, δ is the Kronicker symbol, and $L_n(x)$ is the Laguerre polynomial. Finally, the equation for the self-consistent field reads

$$D(\mathbf{k},\omega)V_{\mathbf{k}}=0, \quad (36)$$

where

$$D(\mathbf{k},\omega)=1-\frac{m\omega_c}{2\pi}v_k\sum_{nn'}H_{nn'}\left[\left(\frac{2}{m\omega_c}\right)^{1/2}k_\perp\right] \times\sum_{\sigma p}\frac{f_\sigma^0(n',p+k_\parallel)-f_\sigma^0(n,p)}{E_{n'\,p+k_\parallel}-E_{n,p}-\omega-i\delta} \quad (37)$$

is the dielectric function in a magnetic field. This function was studied by Akhiezer,[7] and we have merely presented here another method for deriving it, which is in the spirit of the present work. We should mention here a paper by Zyryanov,[8] where a similar method was

employed, but the author failed to obtain the correct dielectric function of Eq. (37), due to some unjustified approximations he made in obtaining the matrix $\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle$. Equation (37) may be also considered as a generalization of Akhiezer's thermal equilibrium result to more general stationary situations.

It is now clear from Eq. (36) what is the origin of the name dielectric function. As usual, the equation $D(\mathbf{k},\omega)=0$ represents the condition for self-consistent longitudinal collective oscillation in a magnetic field. The real part of this equation amounts to the dispersion relation, while the imaginary part of $D(\mathbf{k},\omega)$ represents the generalized Landau damping of plasma waves in a magnetic field. It is not the purpose of our paper to study the properties of the dielectric function, but rather to use it for deriving different results. However, we restrict ourself from now on to the case where $f_\sigma^0(\alpha)$ is such that the collective oscillations are damped in time and do not show instabilities.

B. The Dielectric Tensor

In an electron system, the presence of a magnetic field produces coupling between different components of the current, induced by an electric field, and the components of the latter; that is, electric fields induce currents which are, in general, not parallel to the fields. This is usually expressed by a dielectric tensor $\boldsymbol{\varepsilon}(\mathbf{k},\omega)$ defined by the equation

$$\mathbf{j}(\mathbf{k},\omega)=(i\omega/4\pi)[\boldsymbol{\varepsilon}(\mathbf{k},\omega)-\mathbf{1}]\cdot\mathbf{E}(\mathbf{k},\omega), \tag{38}$$

in terms of the Fourier transforms of the induced current $\mathbf{j}$ and the prevailing field $\mathbf{E}$.

In order to calculate the dielectric tensor, it is convenient to start from the gauge invariant Hamiltonian

$$H=\int d\mathbf{r}\psi_\sigma^\dagger(\mathbf{r}) \times\{(2m)^{-1}[\mathbf{p}-(e/c)\mathbf{A}(\mathbf{r})]^2+e\chi(\mathbf{r})\}\psi_\sigma(\mathbf{r}), \tag{39}$$

where $\mathbf{p}\rightarrow -i(\partial/\partial\mathbf{r})$, $\chi(\mathbf{r})$ is the scalar potential, and the vector potential $\mathbf{A}(\mathbf{r})$ is given as

$$\mathbf{A}(\mathbf{r})=\mathbf{A}_0(\mathbf{r})+\mathbf{A}_1(\mathbf{r}), \tag{40}$$

where $\mathbf{A}_0(\mathbf{r})$ corresponds to the uniform magnetic field $\mathbf{B}$. If we further assume A_1 and χ to be small, the linearized equation for the density matrix takes the form

$$[i(\partial/\partial t)+T_1]f_\sigma^{(1)}(1,1') =[U(1)-U(1')]f_\sigma^{(0)}(1,1'), \tag{41}$$

where T_1 is defined by Eq. (6), $f_\sigma^{(0)}(1,1')$ is given by Eq. (28), and

$$U(\mathbf{r})=-\frac{e}{2mc}\left\{\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0(\mathbf{r})\right]\cdot\mathbf{A}_1(\mathbf{r}) +A_1(\mathbf{r})\cdot\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0(\mathbf{r})\right]\right\}+e\chi(\mathbf{r}). \tag{42}$$

In terms of the Landau's representation, Eq. (41) reads

$$[i(\partial/\partial t)+E_{\alpha'}-E_\alpha]f_\sigma^{(1)}(\alpha,\alpha') =\int d\mathbf{r}_1 d\mathbf{r}_1'\phi_{\alpha'}(\mathbf{r}_1')\phi_\alpha^*(\mathbf{r}_1)[U(\mathbf{r}_1)-U(\mathbf{r}_1')] \times\sum_{\alpha_1}\phi_{\alpha_1}^*(\mathbf{r}_1')\phi_{\alpha_1}(\mathbf{r}_1)f_\sigma^{(0)}(\alpha_1), \tag{43}$$

where ϕ_α is given by Eq. (21).

We turn now to the introduction of the induced charge density and current. The charge density operator

$$\hat{\rho}(\mathbf{r})=e\textstyle\sum_\sigma\psi_\sigma^\dagger(\mathbf{r})\psi_\sigma(\mathbf{r}) \tag{44}$$

yields, for the (induced) charge density,

$$\rho(\mathbf{r}_1)=e\sum_\sigma f_\sigma^{(1)}(1,1') =e\sum_{\sigma\alpha\alpha'}\phi_{\alpha'}^*(\mathbf{r}_1)\phi_\alpha(\mathbf{r}_1)f_\sigma^{(1)}(\alpha,\alpha'), \tag{45}$$

under the assumption that before applying the perturbing field the net charge density vanishes. The Fourier transform of ρ reads

$$\rho(\mathbf{k})=e\sum_{\sigma\alpha\alpha'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle f_\sigma^{(1)}(\alpha,\alpha'). \tag{46}$$

In a similar way, the current operator,

$$\mathbf{j}(\mathbf{r})=\frac{e}{2m}\sum_\sigma\int d\mathbf{r}\psi_\sigma^\dagger(\mathbf{r}^i)\left\{\left[\mathbf{p}'-\frac{e}{c}\mathbf{A}(\mathbf{r}')\right]\delta(\mathbf{r}-\mathbf{r}') +\delta(\mathbf{r}-\mathbf{r}')\left[\mathbf{p}'-\frac{e}{c}\mathbf{A}(\mathbf{r})\right]\right\}\psi_\sigma(\mathbf{r}), \tag{47}$$

yields for the (induced) current

$$\mathbf{j}(\mathbf{k})=\mathbf{j}_0(\mathbf{k})+\mathbf{j}_1(\mathbf{k}), \tag{48}$$

where

$$\mathbf{j}_0(\mathbf{k})=-\frac{e^2}{mc}\sum_{\sigma\alpha}\langle\alpha|e^{i\mathbf{k}\cdot\mathbf{r}}\mathbf{A}_1(\mathbf{r})|\alpha\rangle f_\sigma^{(0)}(\alpha) \tag{49}$$

and

$$\mathbf{j}_1(\mathbf{k})=\frac{e}{m}\sum_{\sigma,\alpha\alpha'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0(\mathbf{r})+\frac{\mathbf{k}}{2}\right]|\alpha\rangle f_\sigma^{(1)}(\alpha,\alpha'), \tag{50}$$

where the definition of the matrix elements is given by Eq. (31), and the assumption of zero net current [for the distribution $f_\sigma^0(\alpha)$] is invoked.

Now, since our linearized equations are gauge invariant, we choose the gauge where $\chi(\mathbf{r})=0$, and the vector potential $\mathbf{A}_1$ is determined by the prevailing electric field

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{k}',\omega)e^{-i\mathbf{k}'\cdot\mathbf{r}+\omega t}, \tag{51}$$

namely

$$\mathbf{A}_1(\mathbf{k}',\omega)=i(c/\omega)\mathbf{E}(\mathbf{k}',\omega). \tag{52}$$

Substituting Eqs. (52) and (51) into Eqs. (42) and (43), we obtain the stationary solution

$$f_\sigma^{(1)}(\alpha,\alpha')=-i\frac{e}{m\omega}\mathbf{E}(\mathbf{k}',\omega)\cdot\langle\alpha'|e^{i\mathbf{k}'\cdot\mathbf{r}}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0+\frac{\mathbf{k}'}{2}\right]|\alpha\rangle^*\frac{f_\sigma^{(0)}(\alpha')-f_\sigma^{(0)}(\alpha)}{E_{\alpha'}-E_\alpha-\omega-i\delta}, \tag{53}$$

where the asterisk stands for the complex conjugate, and δ is a small positive convergent factor. Further, the induced current [Eqs. (49) and (50)] is

$$\mathbf{j}(\mathbf{k},\omega)=-i\frac{e^2N}{m\omega}\mathbf{E}(\mathbf{k},\omega)-i\frac{e^2}{m^2\omega}\mathbf{E}(\mathbf{k}',\omega)\cdot\sum_{\sigma\alpha\alpha'}\langle\alpha'|e^{i\mathbf{k}'\cdot\mathbf{r}}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0+\frac{\mathbf{k}'}{2}\right]|\alpha\rangle^*$$
$$\times\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0+\frac{\mathbf{k}}{2}\right]|\alpha\rangle\frac{f_\sigma^{(0)}(\alpha')-f_\sigma^{(0)}(\alpha)}{E_{\alpha'}-E_\alpha-\omega-i\delta}, \tag{54}$$

where use has been made of Eq. (25). Following a procedure similar to the one which leads from Eq. (32) to Eq. (37), we can write the dyadic

$$\sum_{qq'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0+\frac{k}{2}\right]|\alpha\rangle\langle\alpha|e^{i\mathbf{k}'\cdot\mathbf{r}}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}_0+\frac{\mathbf{k}'}{2}\right]|\alpha\rangle^*=m\omega_c\delta_{\mathbf{k},\mathbf{k}'}\delta_{p',p+k_\parallel}\mathbf{F}_{nn'p}(\mathbf{k})\mathbf{F}_{nn'p}{}^*(\mathbf{k}), \tag{55}$$

where, with $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ the unit vectors in the x, y, z directions,

$$\mathbf{F}_{nn'p}(\mathbf{k})=\int dx e^{ik_xx}\Phi_{n'}\left(x-\frac{k_y}{m\omega_c}\right)\left(-i\mathbf{e}_x\frac{\partial}{\partial x}-\mathbf{e}_ym\omega_cx+\mathbf{e}_zp+\frac{\mathbf{k}}{2}\right)\Phi_n(x), \tag{56}$$

and $\Phi_n(x)$ are the harmonic oscillators' eigenfunctions. Finally, we employ Eq. (38), and with Eqs. (54) and (55), we obtain the dielectric tensor

$$\boldsymbol{\varepsilon}(\mathbf{k},\omega)=\left(1-\frac{\omega_p{}^2}{\omega^2}\right)\mathbf{1}-\frac{\omega_p{}^2}{\omega^2}\frac{\omega_c}{N}\sum_{\sigma nn'p}\mathbf{F}_{nn'p}(\mathbf{k})\mathbf{F}_{nn'p}{}^*(\mathbf{k})\frac{f_\sigma^{(0)}(n',\,p+k_\parallel)-f_\sigma^{(0)}(n,p)}{E_{n',p+k_\parallel}-E_{n,p}-\omega-i\delta}, \tag{57}$$

where $\omega_p{}^2=4\pi e^2N/m$ is the plasma frequency.

We do not intend to study here the properties of the dielectric tensor, but merely to present a method of deriving it from the equations of motion of the density matrix. However, we wish to point out that by using the equation for the continuity of charge, $\partial\rho/\partial t+\partial\mathbf{j}/\partial\mathbf{r}=0$, and the Schrödinger equation [Eq. (16)] one can show that the dielectric function $D(\mathbf{k},\omega)$ of Eq. (37) is, indeed, the longitudinal component $\mathbf{k}\cdot\boldsymbol{\varepsilon}(k,\omega)\cdot\mathbf{k}/k^2$ of Eq. (57).

C. Spectrum of Density Fluctuations

In the theory of linear response of a many-body system, the Fourier transform of the time-dependent density fluctuations, introduced by Van Hove,[16] plays an important role. We wish to present here a simple method of deriving it in the case of electrons in a magnetic field. The method is based on the introduction of the so-called "dressed electrons," i.e., the electrons with their associated clouds.[14]

The spectrum of the electron-density fluctuations is defined by

$$S(\mathbf{k},\omega)=2\,\mathrm{Re}S^+(\mathbf{k},\omega), \tag{58}$$

where

$$S^+(\mathbf{k},\omega)=(2N)^{-1}$$
$$\times\int_0^\infty dte^{i\omega t}\langle n(\mathbf{k},t)n(-\mathbf{k},0)+n(-\mathbf{k},0)n(\mathbf{k},t)\rangle, \tag{59}$$

Re stands for the real part, $n(\mathbf{k},t)$ is the spatial Fourier transform of the electron-density operator, and the $\langle\cdots\rangle$ is taken in the sense of Eq. (2). A standard method of calculating Eq. (59) is the Green's function technique, which is limited to thermal equilibrium (and zero temperature), and it is essentially given by Akhiezer[7] in his study of the problem of the energy loss of a charged particle moving through the system, while in thermal equilibrium. Our approach to the problem, which makes use of the "dressed particle" method, covers a wider range of situations, namely, systems in stationary states of the one-particle distribution function.

Consider a test particle, with the same properties of

[16] L. Van Hove, Phys. Rev. **95**, 249 (1954).

the electrons, embedded into our system. If we assign to the test particle, in the given magnetic field, the Landau states β and β', it will interact with the electrons of the system, via the potential (energy)

$$V_{\text{test}}(\mathbf{k},t)=v_k\langle\beta'|e^{i\mathbf{k}\cdot\mathbf{r}}|\beta\rangle a_{\beta'}{}^{\dagger}a_\beta e^{i(E_{\beta'}-E_\beta)t}, \quad (60)$$

where $\alpha_\beta{}^{\dagger}$, a_β are the creation and annihilation operators of the test particle, and E_β is the associated energy. Assuming the system to respond linearly to the small perturbation caused by the test particle, we can write the equation for the density matrix as

$$\left(i\frac{\partial}{\partial t}+E_{\alpha'}-E_\alpha\right)f_\sigma{}^{(1)}(\alpha,\alpha')$$
$$=[f_\sigma{}^{(0)}(\alpha')-f_\sigma{}^{(0)}(\alpha)]\sum_k v_k\langle\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha'\rangle$$
$$\times\{\sum_{\sigma\alpha_1\alpha_1'}\langle\alpha_1'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha_1\rangle f_\sigma{}^{(1)}(\alpha_1,\alpha_1')$$
$$+\langle\beta'|e^{i\mathbf{k}\cdot\mathbf{r}}|\beta\rangle a_{\beta'}{}^{\dagger}a_\beta e^{i(E_{\beta'}-E_\beta)t}\}, \quad (61)$$

where we have skipped over some steps, which are similar to the derivation of the dielectric function. The stationary cloud associated with the test particle is, thus, given by

$$n^{(1)}(\mathbf{k}\,t)=\sum_{\sigma\alpha\alpha'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle f_\sigma{}^{(1)}(\alpha,\alpha'), \quad (62)$$

which yields, upon solving Eq. (61),

$$n^{(1)}(\mathbf{k},t)=\frac{m\omega_c}{2\pi}\frac{v_k}{D(\mathbf{k},E_{\beta'}-E_\beta)}\sum_{nn'}H_{nn'}\left[\left(\frac{2}{m\omega_c}\right)^{1/2}k_\perp\right]$$
$$\times\sum_{\sigma p}\frac{f_\sigma{}^{(0)}(n',p+k_{\parallel})-f_\sigma{}^{0}(n,p)}{E_{\alpha'}-E_\alpha-E_{\beta'}+E_\beta-i\delta}$$
$$\times\langle\beta'|e^{i\mathbf{k}\cdot\mathbf{r}}|\beta\rangle a_{\beta'}{}^{\dagger}a_\beta e^{i(E_{\beta'}-E_\beta)t}, \quad (63)$$

where D and H are given by Eqs. (37) and (34) respectively, and $f_\sigma{}^{0}(n,p)$ is the stationary "diagonal" distribution function of the (field) electrons.

We now consider the test particle with its associated cloud as a "dressed electron" with the assigned Landau states β' and β; hence the corresponding density operator reads

$$n(\mathbf{k},t)=\langle\beta'|e^{i\mathbf{k}\cdot\mathbf{r}}|\beta\rangle a_{\beta'}{}^{\dagger}a_\beta e^{i(E_{\beta'}-E_\beta)t}+n^{(1)}(\mathbf{k},t)$$
$$=\frac{\langle\beta'|e^{i\mathbf{k}\cdot\mathbf{r}}|\beta\rangle}{D(\mathbf{k},E_{\beta'}-E_\beta)}e^{i(E_{\beta'}-E_\beta)t}a_{\beta'}{}^{\dagger}a_\beta. \quad (64)$$

If we substitute Eq. (64) into Eq. (59), consider the average $\langle\cdots\rangle$ as taken over "free" dressed particles, and remember that we deal with fermions, the spectrum fluctuation reads

$$S(\mathbf{k},\omega)=\frac{m\omega_c}{2N}\sum_{nn'}\frac{H_{nn'}[(2/m\omega_c)^{1/2}k_\perp]}{|D(\mathbf{k},\omega)|^2}$$
$$\times\sum_{p,\sigma}\delta(E_{n'\,p+k_{\parallel}}-E_{n,p}-\omega)$$
$$\times\{f_\sigma{}^{(0)}(n',p+k_{\parallel})[1-f_\sigma{}^{(0)}(n,p)]$$
$$+f_\sigma{}^{(0)}(n,p)[1-f_\sigma{}^{(0)}(n',p+k_{\parallel})]\}. \quad (65)$$

This is our result for the spectrum of density fluctuation in a uniform magnetic field, in the self-consistent field approximation. Its wide variety of applications is well known and we do not consider them here.

IV. KINETIC EQUATION

In the present section we consider the problem of deriving a kinetic equation for the electrons in a uniform magnetic field.[16a] This task is accomplished by solving Eq. (14) for the correlation function g in terms of the $f_\sigma(1,1')$, and substituting the result into Eq. (13) to yield an equation for f_σ itself. We shall follow closely the approach investigated in I, with appropriate modifications to suit the present problem.

The first step is the transformation of Eqs. (13) and (14) to the proper Landau representation. We restrict ourselves to a derivation of a kinetic equation for the distribution function $f_\sigma(n,p)$ of the electrons among the states specified by $E_{n,p}$. Thus, we assume that in Eq. (11) f_σ is represented by

$$f_\sigma(1,1')=\sum_\alpha\phi_\alpha{}^{*}(\mathbf{r}_1')\phi_\alpha(\mathbf{r}_1)f_\sigma(\alpha), \quad (66)$$

while g takes care of the "off-diagonal" elements, namely

$$g_{\sigma_1\sigma_2}(1,2;1',2')=\sum_{\alpha_1\alpha_1'\alpha_2\alpha_2'}\phi_{\alpha_1'}{}^{*}(\mathbf{r}_1')\phi_{\alpha_2'}{}^{*}(\mathbf{r}_2')$$
$$\times\phi_{\alpha_2}(\mathbf{r}_2)\phi_{\alpha_1}(\mathbf{r}_1)g_{\sigma_1\sigma_2}(\alpha_1,\alpha_2;\alpha_1',\alpha_2') \quad (67)$$

without restricting *a priori* the dependence of $g_{\sigma_1\sigma_2}(\alpha_1,\alpha_2;\alpha_1',\alpha_2')$ on the α's. This scheme of approximation will be justified *a posteriori* by the explicit dependence of the correlation function on the strength of the interaction

[the ratio= (potential energy)/(kinetic energy)].

Equation (13) now reads

$$(\partial/\partial t)f_\sigma(n,\rho)=-2\sum_k v_k\sum_{q,\alpha'}\text{Im}[G_\sigma(\alpha,\alpha';\mathbf{k})], \quad (68)$$

where v_k is the Fourier transform of $v(r)$, Im stands for

[16a] We are thankful to Dr. E. Klevans for pointing out to us an unpublished work of V. P. Silin [Salzberg Conference, Paper CN-10/247, September 1961, English transl.: AEC-Tr 5589, 1963 (unpublished)], where the kinetic equation has been investigated by employing a different method.

the imaginary part, and

$$G_{\sigma_1}(\alpha_1,\alpha_1';\mathbf{k})=\sum_{\sigma_2\alpha_2\alpha_2'}\langle\alpha_1'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha_1\rangle \times\langle\alpha_2'|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha_2\rangle g_{\sigma_1\sigma_2}(\alpha_1,\alpha_2;\alpha_1'\,\alpha_2). \quad (69)$$

In deriving Eq. (68) we have used the relation

$$g_{\sigma_1\sigma_2}{}^*(\alpha_1,\alpha_2;\alpha_1',\alpha_2')=g_{\sigma_1\sigma_2}(\alpha_1',\alpha_2';\alpha_1,\alpha_2). \quad (70)$$

Similarly, Eq. (14) takes the form

$$\begin{aligned}&[i(\partial/\partial t)+E_{\alpha_1'}-E_{\alpha_1}+E_{\alpha_2'}-E_{\alpha_2}]g_{\sigma_1\sigma_2}(\alpha_1,\alpha_2;\alpha_1',\alpha_2')\\&\quad-\sum_{\mathbf{k}'}v_{\mathbf{k}'}\langle\alpha_1|e^{i\mathbf{k}'\cdot\mathbf{r}}|\alpha_1'\rangle[F_{\sigma_1}(\alpha_1,\alpha_1')-F_{\sigma_1}(\alpha_1',\alpha_1)]\sum_{\sigma_3\alpha_3\alpha_3'}\langle\alpha_3'|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha_3\rangle g_{\sigma_2\sigma_3}(\alpha_2,\alpha_3;\alpha_2',\alpha_3')\\&\quad-\sum_{\mathbf{k}'}v_{k'}\langle\alpha_2|e^{i\mathbf{k}'\cdot\mathbf{r}}|\alpha_2'\rangle[F_{\sigma_2}(\alpha_2,\alpha_2')-F_{\sigma_1}(\alpha_2',\alpha_2)]\sum_{\sigma_3\alpha_3\alpha_3'}\langle\alpha_3'|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha_3\rangle g_{\sigma_3\sigma_1}(\alpha_1\alpha_3;\alpha_1',\alpha_3')\\&\quad=\sum_{\mathbf{k}'}v_{k'}\langle\alpha_1|e^{i\mathbf{k}'\cdot\mathbf{r}}|\alpha_1'\rangle\langle\alpha_2|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha_2'\rangle[F_{\sigma_1}(\alpha_1,\alpha_1')F_{\sigma_2}(\alpha_2,\alpha_2')-F_{\sigma_1}(\alpha_1',\alpha_1)F_{\sigma_2}(\alpha_2',\alpha_2)], \quad (71)\end{aligned}$$

where

$$F_\sigma(\alpha,\alpha')=f_\sigma(\alpha')[1-f_\sigma(\alpha)]. \quad (72)$$

We now notice that in order to obtain the kinetic equation it is enough to solve Eq. (71) only for the expression

$$\mathrm{Im}\sum_{qq'}G_\sigma(\alpha\,\alpha';\mathbf{k}).$$

Hence we define a function

$$\mathcal{G}(\beta_1,\beta_2;\mathbf{k})=\sum_{q_1q_1'q_2q_2'}\langle\alpha_1'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha_1\rangle \times\langle\alpha_2'|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha_2\rangle g_{\sigma_1\sigma_2}(\alpha_1,\alpha_2;\alpha_1',\alpha_2'), \quad (73)$$

where β stands for the shorthand

$$\beta\equiv\sigma;n,p;n',p'. \quad (74)$$

Using Eq. (71) we can write

$$[i(\partial/\partial t)+K(\beta_1,\mathbf{k})+K(\beta_2,-\mathbf{k})]\mathcal{G}(\beta_1,\beta_2;\mathbf{k}) =\mathcal{S}(\beta_1,\beta_2;\mathbf{k}), \quad (75)$$

where $K(\beta,\mathbf{k})$ is the integral operator

$$K(\beta,\mathbf{k})=E_{\alpha'}-E_\alpha-\frac{m\omega_c}{2\pi}v_k\delta_{p',p+k_\parallel}H_{nn'}\left[\left(\frac{2}{m\omega_c}\right)^{1/2}k_\perp\right] \times[F_\sigma(\alpha,\alpha')-F_\sigma(\alpha',\alpha)]\sum_{\sigma,nn',pp'}, \quad (76)$$

and the source term

$$\begin{aligned}&\mathcal{S}(\beta_1,\beta_2;\mathbf{k})\\&=(m\omega_c/2\pi)^2v_k\delta_{p_1',p_1+k_\parallel}\delta_{p_2',p_2-k_\parallel}\\&\quad\times H_{n_1n_1'}[(2/m\omega_c)^{1/2}k_\perp]H_{n_2n_2'}[(2/m\omega_c)^{1/2}k_\perp]\\&\quad\times[F_{\sigma_1}(\alpha_1,\alpha_1')F_{\sigma_2}(\alpha_2,\alpha_2')-F_{\sigma_1}(\alpha_1',\alpha_1)F_{\sigma_2}(\alpha_2',\alpha_2)].\end{aligned} \quad (77)$$

In obtaining Eq. (75) we have made use of the symmetry property

$$g_{\sigma_1\sigma_2}(\alpha_1,\alpha_2;\alpha_1',\alpha_2')=g_{\sigma_2\sigma_1}(\alpha_2,\alpha_1;\alpha_2',\alpha_1').$$

Adopting the procedure employed in I, which is originally due to Dupree,[15] we introduce an operator $\mathcal{P}(\beta,\mathbf{k},t)$ by the equation

$$[i(\partial/\partial t)+K(\beta,\mathbf{k})]\mathcal{P}(\beta,\mathbf{k},t)=0, \quad (78)$$

with the initial value $\mathcal{P}(\beta,\mathbf{k},t=0)=I$, where I is the unity operator. We also invoke the *adiabatic hypothesis*, which is due to Bogoliubov,[17] that the correlation function of the two electrons reaches an asymptotic value in a time short compared with the time in which the distribution function changes appreciably. Noticing that $K(\beta_1,\mathbf{k})$ commutes with $K(\beta_2,-\mathbf{k})$ and introducing Laplace transforms of Eq. (78), we can write the solution of Eq. (75) as

$$\mathcal{G}(\beta_1,\beta_2;\mathbf{k})=-i\int_0^\infty d\tau e^{-\epsilon\tau}\int_{c_1}\frac{ds_1}{2\pi i}e^{s_1\tau}\int_{c_2}\frac{ds_2}{2\pi i}e^{s_2\tau} \times\mathcal{P}(\beta_1,\mathbf{k},s_1)\mathcal{P}(\beta_2,-\mathbf{k},s_2)\mathcal{S}(\beta_1,\beta_2;\mathbf{k}), \quad (79)$$

where ϵ is a small positive convergence factor, and c_1, c_2 are the usual Laplace transform inversion contours. The operator $\mathcal{P}$ is obtained by solving Eq. (79), i.e.,

$$\mathcal{P}(\beta,\mathbf{k},s)=i\frac{1}{is+\Delta(\beta)}\left\{I+\frac{A(\beta,\mathbf{k})}{D(\mathbf{k},s)}\sum_\beta\frac{I}{is+\Delta(\beta)}\right\}, \quad (80)$$

where

$$\Delta(\beta)=E_{\alpha'}-E_\alpha$$

$$A(\beta,\mathbf{k})=\frac{m\omega_c}{2\pi}v_k\delta_{p',p+k_\parallel}H_{nn'}\left[\left(\frac{2}{m\omega_c}\right)^{1/2}k_\perp\right] \times[F_\sigma(\alpha,\alpha')-F_\sigma(\alpha',\alpha)], \quad (80)$$

and

$$D(\mathbf{k},s)=1-\sum_\beta\frac{A(\beta,\mathbf{k})}{is+\Delta(\beta)}. \quad (81)$$

[17] N. N. Bogoliubov, in *Studies of Statistical Mechanics*, edited by J. deBoer and G. E. Uhlenbech (North-Holland Publishing Company, Amsterdam, 1962).

Finally, after some algebraic manipulations (see I for details) we obtain

$$\mathrm{Im}[\sum_{q_1 q_1'} G_{\sigma_1}(\alpha_1,\alpha_1';\mathbf{k})]=\mathrm{Im}\int_{-\infty}^{\infty}\frac{dw}{2\pi i}\frac{1}{\Delta(\beta_1)-w+i\delta}\left[1+\frac{A(\beta_1,\mathbf{k})}{D(\mathbf{k},w)}\sum_{\beta_1}\frac{1}{\Delta(\beta_1)-w+i\delta}\right]$$
$$\times\frac{1}{D^*(k,w)}\sum_{\beta_2}\frac{1}{\Delta(\beta_2)+w+i\delta}\mathcal{S}(\beta_1,\beta_2;\mathbf{k}), \quad (82)$$

where the limit $\delta\to+0$ is understood, and $D(\mathbf{k},w)$ is given by Eq. (37). Evaluating the w integration in Eq. (82) we obtain our final result

$$\frac{\partial}{\partial t}f_\sigma(n,p)=-2\pi\sum_{k,p',\sigma'}\sum_{n_1,n_2,n_3}\frac{v_k^2 H_{nn_1}[(2/m\omega_c)^{1/2}k_\perp]H_{n_2n_3}[(2/m\omega_c)^{1/2}k_\perp]}{|D(\mathbf{k},E_{n_1,p+k_\parallel}-E_{n,p})|^2}$$
$$\times\delta(E_{n_1,p+k_\parallel}-E_{n,p}+E_{n_3,p'-k_\parallel}-E_{n_2,p'})\{f_\sigma(n_1,p+k_\parallel)f_{\sigma'}(n_3,p'-k_\parallel)[1-f_\sigma(n,p)][1-f_{\sigma'}(n_2,p')]$$
$$-f_\sigma(n,p)f_{\sigma'}(n_2,p')[1-f_\sigma(n_1,p+k_\parallel)][1-f_{\sigma'}(n_3,p'-k_\parallel)]\}. \quad (83)$$

Equation (83) is the kinetic equation for the electrons in a magnetic field. The individual and collective aspects of the electron interactions in a magnetic field in the self-consistent field approximation are included. The collective aspects are represented as a dynamic screening by the dielectric function D in the denominator. The latter is due to the simultaneous motion of the interacting electrons in the magnetic field. It is obvious, by inspection, that the Fermi distribution

$$f_\sigma(n,p)=\{\exp[T^{-1}(E_{n,p}-\mu)]+1\}^{-1} \quad (84)$$

is a stationary equilibrium solution of Eq. (83), and therefore Eq. (83) can be regarded as governs the approach to equilibrium of the system of electrons. In Eq. (84) T and μ are, respectively, the temperature in energy units and the chemical potential of the electrons.

V. DISCUSSION

In the present paper we have studied some aspects of the correlations between interacting electrons in a strong magnetic field. We have used a quantum kinetic approach, taking into account the fact that the orbital motion of the electrons in a magnetic field is quantized. This is accomplished by writing the density matrix for one and two electrons in the so-called "Landau representation." Starting from the equations of motion of the second quantization operators, a hierarchy of equations for the density matrices have been introduced. This hierarchy is "naturally" truncated in the self-consistent field approximation. Thus the magnetic field is considered as of zeroth order in the equations of motion for the electrons, while the interactions between the electrons is regarded as a first-order effect. As usual, the fact that each electron "sees" many others, due to the long range of the Coulomb potential, changes the order of magnitude of the interaction terms in the equations when many particles are involved (i.e., when an integration over the long-range potential is present and the exchange effects do not cut it off). In this way the SCF method takes proper account of both the individual and collective aspects of the electron interactions.

While the electrons are moving in their quantized orbits in the magnetic field, they interact with each other and develop correlations. This is expressed to a large extent by the dielectric function calculated in Sec. III. The dielectric function reflects the two main effects of the correlations, i.e., the collective excitations of the system and the dynamical shielding. The spectrum of excitations is given by the zeros of the dielectric function, while the shielding is expressed by singling out a "test particle" and "dressing" it [Eq. (64)]. This dielectric function plays a role also in the equation governing the approach to equilibrium of the system. The electrons approach thermal equilibrium as though they are "dressed" by each other, as shown in Eq. (23). If an external field is present, the induced current is related to the field by a conductivity tensor, due to the magnetic field, and a dielectric tensor expresses the response of the system to this external field.

APPENDIX

We wish to present here the derivation of Eq. (33). Using Eqs. (21) and (31) we write

$$z=\sum_{qq'}\langle\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha\rangle\langle\alpha|e^{-i\mathbf{k}'\cdot\mathbf{r}}|\alpha'\rangle$$
$$=(2\pi)^{-4}\sum_{qq'}\int d\mathbf{r}d\mathbf{r}'e^{-ip'z-iq'y}\Phi_{n'}(x-q'/m\omega_c)e^{i\mathbf{k}\cdot\mathbf{r}}e^{ipz+iqy}\Phi_n(x-q/m\omega_c)$$
$$\times e^{-ipz'-iqy'}\Phi_n(x'-q/m\omega_c)e^{-i\mathbf{k}'\cdot\mathbf{r}'}e^{ip'z'+iq'y'}\Phi_{n'}(x'-q'/m\omega_c). \quad (A1)$$

The integration over z, z', y, and y' is straightforward and leads to

$$z=\delta_{k_z,k_z'}\delta_{k_y,k_y'}\delta_{p',p+k_z}\sum_{qq'}\delta_{q',q+k_y}\int dx dx' e^{ik_x x-ik_x' x'}\Phi_{n'}(x-q'/m\omega_c)$$
$$+\Phi_n(x-q/m\omega_c)\Phi_n(x'-q/m\omega_c)\Phi_{n'}(x'-q'/m\omega_c). \quad \text{(A2)}$$

Under the transformation

$$x=R+\xi/2, \quad x'=R-\xi/2, \quad q=-Q+m\omega_c R, \quad q'=-Q'+m\omega_c R, \quad \text{(A3)}$$

and an integration over R, Eq. (A2) becomes

$$z=\delta_{\mathbf{k}',\mathbf{k}}\delta_{p',p+k_z}\sum_{QQ'}\delta_{Q',Q'-k_y}\int d\xi e^{ik_x\xi}\Phi_{n'}(Q'/m\omega_c+\xi/2)\Phi_{n'}(Q'/m\omega_c-\xi/2)\Phi_n(Q/m\omega_c+\xi/2)\Phi_n(Q/m\omega_c-\xi/2). \quad \text{(A4)}$$

Notice the Kronicker δ for $\mathbf{k}'$ and $\mathbf{k}$, which corresponds to invariance under spatial translation. Changing the summation by an integration, and using an integral representation for the δ function, we obtain

$$z=\delta_{\mathbf{k}',\mathbf{k}}\delta_{p',p+k_\parallel}(m\omega_c/2\pi)^2\int d\xi d\eta e^{ik_x\xi+ik_y\eta}\int du e^{im\omega_c\eta u}\Phi_n(u+\xi/2)\Phi_n(u-\xi/2)$$
$$\times\int du' e^{im\omega_c\eta u'}\Phi_{n'}(u'+\xi/2)\Phi_{n'}(u'-\xi/2)$$
$$=\delta_{\mathbf{k}',\mathbf{k}}\delta_{p',p+k_\parallel}\left(\frac{m\omega_c}{2\pi}\right)^2\int d\xi d\eta e^{ik_x\xi+ik_y\eta}e^{-m\omega_c(\xi^2+\eta^2/2)}L_n\left(m\omega_c\frac{\xi^2+\eta^2}{2}\right)L_{n'}\left(m\omega_c\frac{\xi^2+\eta^2}{2}\right), \quad \text{(A5)}$$

where use has been made of the equation

$$\int du e^{ia\eta u}\Phi_n(u+\xi/2)\Phi_n(u-\xi/2)=e^{-a\rho^2/2}L_n(a\rho^2/2), \quad \text{(A6)}$$

$$\rho=\xi^2+\eta^2,$$

with $L_n(x)$ the Laguerre polynomial. Finally, if we set

$$\xi=(2/m\omega_c)^{1/2}x, \quad \eta=(2/m\omega_c)^{1/2}y, \quad \text{(A7)}$$

and use circular coordinates for x, y, k_x, and k_y

$$x=r\cos\vartheta, \quad y=r\sin\vartheta, \quad k_x=k_\perp\cos\alpha, \quad k_y=k_\perp\sin\alpha, \quad \text{(A8)}$$

we obtain

$$z=\delta_{\mathbf{k}',\mathbf{k}}\delta_{p',p+k_\parallel}\frac{m\omega_c}{(2\pi)^2}\int_0^\infty dr 2re^{-r^2}L_n(r^2)L_{n'}(r^2)\int_0^{2\pi}d\vartheta e^{i(2/m\omega_c)^{1/2}k_\perp r\cos(\vartheta-\alpha)}, \quad \text{(A9)}$$

and setting $s=r^2$ yields

$$z=\delta_{\mathbf{k}',\mathbf{k}}\delta_{p',p+k_\parallel}\frac{m\omega_c}{2\pi}\int ds I_0\left[\left(\frac{2}{m\omega_c}\right)^{1/2}(s)^{1/2}k_\perp\right]e^{-s}L_n(s)L_{n'}(s), \quad \text{(A10)}$$

as stated in Eq. (33).